Results of the Adjustment of the United European Levelling Network 1995 (UELN-95/98)

— Report by the UELN data centre —

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1. Introduction

Since 1994 the work at the UELN was continued under the name UELN-95 after a break of 10 years. In accordance with Resolution No. 3 of the EUREF Symposium 1994 in Warsaw, the objective of the UELN project is to establish a Unified Vertical Datum for Europe at the one-decimeter level with the simultaneous extension of UELN as far as possible to the Eastern European countries.

With this report the results of the UELN-95/13 adjustment are handed over to the participating countries under the name UELN-95/98 as decided at the last EUREF Symposium 1998 in Bad Neuenahr/Ahrweiler.

2. General Development since 1994

In summary the UELN-95 development is marked by the following adjustment steps (figure 1):

- UELN-73/94: Repetition of the UELN-73/86 adjustment using weights for the single blocks derived from the variance component estimation
- UELN-95/1, -95/2: The previous network block of Germany was removed from the UELN-73 and replaced by the new network block of the DHHN92
- UELN-95/3: The previous network block of Austria was replaced by the remeasurement of the Austrian First Order Levelling Network
- UELN-94/4, -95/5, -95/6, -95/7, -95/8: Integration of the new national network blocks of the Czech Republic (-95/4), Hungary (-95/5), Slovenia (-95/6), Poland (-95/7) and Slovakia (-95/8)
- UELN-95/9, -95/10: The previous network block of the Netherlands was replaced by the Fourth Precise Levelling Network of the Netherlands
- UELN-95/11: The previous network of Denmark was replaced by the new remeasurement of the Danish First Order Levelling Network
- UELN-95/12: Integration of the new national network blocks of the Republic of Bosnia and Herzegovina, the Republic of Croatia, Monte Negro, the Republic of Slovenia and Vojvodina
- UELN-95/13 (named as UELN-95/98): Integration of some additional measurements in the Austrian First Order Levelling Network; addition of two border connections between Croatia and Hungary as well as one connection between Slovenia and Italy. The final network design of UELN-95/98 is shown in figure 2.

The extension to UELN-95 was performed in two qualitatively different steps:

- Substitution of data material of such network blocks which were already part of UELN-73 but show current new measurements with improved (mostly more dense) network configuration (intensive enlargement)
- Adding new national network blocks of Central and Eastern Europe which were not part of UELN-73 (extensive enlargement)

3. On the Method of Adjusting UELN-95

The program system HOENA, developed at BKG (IfAG) (SCHOCH, 1995), was used for the adjustment of the UELN-95. The programs of HOENA are written in FORTRAN77 and can be operated under UNIX.

The adjustment of geopotential numbers was performed as an unconstrained adjustment linked to the reference point of UELN-73 (gauge Amsterdam).

Some countries delivered their levelling networks with all intermediate points of the levelling lines. In these cases the nodal points and the relations between them were determined by an auxiliary program before the beginning of the adjustment.

Height networks with up to 5000 points can be adjusted. The files of the observation type „measured height differences„ can contain an arbitrary number of relations. The variance component estimation is designed for maximally 50 observation groups.

For data organization and management, an INFORMIX data bank for height networks is used.

The a-priori standard deviation of the geopotential differences was derived as usual from the square root of the levelling distance in km (see data file 4 of the country files). The weights in data file 3 are the final weights after the variance component estimation, i. e. the starting weights were multiplied by a factor, which was a constant for one observation group and resulted from the variance component estimation.

4. Calculation of Normal Heights

Gravity-related heights are defined in general by

$$\frac{C}{G}$$

where *G* is a suitable gravity value. The kind of heights differs by the kind of gravity value *G* in the denominator.

The results of the UELN adjustment are given in geopotential numbers. The geopotential number *C* is determined by levelling observation Δh and gravity measurements *g* from

$$C = \Sigma g \Delta h.$$

The formula for the normal heights is

$$\frac{C}{\bar{\gamma}}$$

where $\bar{\gamma}$ is the average value of the normal gravity along the normal plumb line between the ellipsoid and the telluroid and between the quasigeoid and the earth surface respectively.

The normal gravity γ_0 at the level spheroid, which is needed for computing the average value of the normal gravity $\bar{\gamma}$, was computed by the formula for γ_0 in the GRS80 (MORITZ, 1988):

$$\gamma_0 = \gamma_e \frac{1 + k \cdot \sin^2 \varphi}{\sqrt{1 - e^2 \sin^2 \varphi}}$$

with

$$\gamma_e = 9.7803267715 \mathrm{m} \cdot \mathrm{s}^{-2}$$
$$k = 0.001931851353$$
$$e^2 = 0.0066943800229$$
$$\varphi = \text{geographical latitude (in the ETRS89)}$$

The average value of the normal gravity along the normal plumb line was determined by

$$\bar{\gamma} \approx \gamma_m = \gamma_0 - \frac{0.3086 \mathrm{mgal/m} \cdot h}{2} + \frac{0.072 \cdot 10^{-6} \mathrm{mgal/m^2} \cdot h^2}{2} \cdot .$$

The required accuracy of the latitude is dependent on the height of the points.

Up to a height of 500 m the latitude is needed with an accuracy of 10“, up to the height of 1500 m an accuracy of 3“ is needed in order to get the normal height with an accuracy of 0.1 mm. The coordinates supplied by the participating countries were transformed into the ETRS89.

5. Results

Table 1 shows the results of the variance component estimation of the adjustment variant UELN-95/13.

Block	Number of observations	Sum of redundancies	a posteriori standard deviation [kgal • mm]
Austria	145	38.943	0.80
Belgium	54	19.488	1.22
Switzerland	13	4.459	1.06
Germany	755	271.808	0.85
Denmark	1036	312.874	0.59
Spain	101	27.254	1.85
France	175	46.857	2.01
Italy	97	33.018	1.76
Netherlands	932	163.933	1.08
Portugal	22	5.857	1.77
Great Britain	60	15.000	1.72
Norway	194	70.992	1.67
Finland	89	20.142	0.76
Sweden	122	34.865	1.74
Czech Republic	82	28.388	1.10
Hungary	54	13.472	0.52
Poland	179	54.777	0.99
Slovakia	74	18.467	1.41
Bosnia/Herzegovina, Croatia, Monte Negro Slovenia, Vojvodina	79	19.405	0.90
Total:	4263	1200.000	

Table 1: Accuracy of the UELN-95/13 adjustment after variance component estimation with status of November, 1998 (Number of unknowns: 3063, Number of observations: 4263, Number of degrees of freedom: 1200).

Table 2 shows the development of the statistical data after the individual steps of the adjustment.

	Adjustment variants UELN-													
Type of data	73/94	95/1	95/2	95/3	95/4	95/5	95/6	95/7	95/8	95/9	95/10	95/11	95/12	95/13
Number of unknowns	449	880	1188	1256	1309	1342	1350	1467	1531	2160	2307	2994	3041	3063
Number of observations	619	1122	1722	1833	1915	1960	1971	2142	2232	2988	3172	4166	4229	4263
Number of degrees of freedom	170	322	574	577	606	618	621	675	701	828	865	1172	1188	1200
Redundancy, average	0.275	0.287	0.324	0.315	0.315	0.315	0.315	0.315	0.314	0.277	0.273	0.281	0.281	0.281
Standard deviation of adjusted geopotential differences [kgal • mm]: (mean value)	12.79	12.08	9.71	9.48	9.35	9.27	9.24	9.03	8.93	7.81	7.58	6.94	6.64	6.62
Standard deviation of a posteriori weight unit referred to a levelling distance of 1 km [kgal • mm]:	1.70	1.61	1.33	1.31	1.30	1.29	1.29	1.26	1.26	1.23	1.23	1.10	1.10	1.10

The example of the new German network block in the variants UELN-95/1 and UELN-95/2 shows clearly how a fine meshed nodal point network influences the redundancy and the accuracy measures.

The continuous increase of the accuracy from the variant UELN-95/3 to UELN-95/8, noticeable in the falling standard deviations of the adjusted geopotential number differences, was caused above all by the high quality of the new integrated network blocks of Eastern Europe.

The integration of the new network block of the Netherlands caused an obvious increase of the accuracy because of the improvement of the connection of the UELN-95 to the reference point gauge Amsterdam. The increase in accuracy is mainly in the direction of those network blocks into which the national levelling networks with high density and homogeneity (east-southeast) were introduced.

That should be a reason for the participating countries of Western Europe to replace their networks in the future by the complete recent national first order levelling networks.

The error propagation of the UELN-95/98 starting from the reference tide gauge Amsterdam is shown in figure 3.

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