IAG Sub-commission 1.3a EUREF

Conventions for the Definition and Realization of a European Vertical Reference System (EVRS) – EVRS Conventions 2007 –

Johannes Ihde[1], Jaakko Mäkinen[2], Martina Sacher[1],

Preamble / Foreword for a special edition EVRS

During the last ten years significant progress has been achieved in the definition and realization of the European Vertical Reference System, especially by the IAG Subcommission EUREF (see http://crs.bkg.bund.de/evrs). After four years' work, the solution of the United European Levelling Network (UELN) 95/98 was distributed to the 20 participating countries in the beginning of 1999. One year later at the EUREF Symposium 2000 in Tromsø the conventions for the European Vertical Reference System 2000 (EVRS2000) were approved, and the realization, the European Vertical Reference 2000 (EVRF2000) based on the UELN 95/98 solution was adopted (see IHDE, J., AUGATH, W., 2000 and 2002, RUMMEL, R., HECK, B., 2000)

The need for the harmonisation of the vertical reference of spatial coordinates was raised at the 9th EC GI-GIS conference on 24-26 June 2003 in La Coruña. It is driven by the fact that exact knowledge, understanding, management, and subsequent processing of the spatial coordinates of any European GI dataset is one of the central aspects of cross-border GI interoperability.

The adoption of a common European Vertical Reference System is a necessary condition for a future simplification in data harmonisation and interoperability.

At a workshop on "Vertical Reference Systems for Europe", held on April 5th to 7th, 2004 in Frankfurt am Main, the needs, requirements, and problems in establishing a pan-European Vertical Reference System were debated to fulfil scientific and practical requirements for vertical georeferencing of the next decades. This workshop was jointly organised by the Joint Research Centre (JRC) of the European Commission and by EuroGeographics, Expert Group Geodesy with support from the IAG Sub-Commission for Europe EUREF.

At the EUREF 2005 Symposium in Vienna the need for a new realization of the EVRS was considered. At the EUREF Symposium 2007 in London the Technical Working Group (TWG) of the EUREF was asked to prepare the technical specifications (conventions) for a new EVRS and its realisation (EVRF2007).

Scope/Summary

The basic concept and the conventions for the definition and 2007 realization of the European Vertical Reference System (EVRS) are presented. The EVRS realization 2007 (EVRF2007) is a continuation of the United European Levelling Network solution UELN 95/98 under consideration of the development of the European Combined Geodetic Network (ECGN) and the realization of a vertical reference system on a global level.

We expect that the conventions will be used by the scientific geodetic community, by national mapping agencies, and by commercial service providers for stationary, kinematic and dynamic tasks in vertical positioning.

The objectives of EVRS realization 2007, which is named EVRF2007, are:

[1] Bundesamt für Kartographie und Geodäsie, Richard-Strauss-Allee 11, D-60598 Frankfurt am Main, Germany
[2] Finnish Geodetic Institute, Department of Geodesy and Geodynamics, Geodeetinrinne 2, FIN-02431 Masala

- to fulfil the EU requirements for seamless, harmonised vertical data,
- to prepare recommendations to the European Commission for a future adoption of a common European Vertical Reference System to be proposed in the INSPIRE (Infrastructure for Spatial Information in Europe) Directive,
- to provide European users and producers of height information with a vertical reference system, which is based on up-to-date datasets and on advanced conventions for EVRS definition and realization.

With the EVRF2007 a Europe-wide vertical reference with sub-decimetre accuracy will be possible. The European Gravimetric Geoid EGG 2007 shall be related to the ETRS89 and EVRF2007.

1. Introduction

With the European Vertical Reference Frame 2000 (EVRF2000), a realization of the European Vertical Reference System 2000 (EVRS2000), a pan-European sub-metre accuracy level was achieved which satisfies ordinary GIS users' requirements. The practical use is supported by the publication of parameters that describe the national vertical reference systems and their relationships to the EVRF2000.

EVRS2000 is a gravity-related height reference system. It was defined by the following conventions (see IHDE, J., AUGATH, W., 2000):

a) The vertical datum is the zero level on which the Earth gravity field potential W_0 is equal to the normal potential of the mean Earth ellipsoid U_0.

$$W_0 = U_0 .$$

b) The height components are the differences ΔW_P between the potential W_P of the Earth gravity field through the considered points P and the potential of the EVRS zero level W_0. The potential difference $-\Delta W_P$ is also designated as geopotential number c_P:

$$-\Delta W_P = W_0 - W_P = c_P .$$

Normal heights are equivalent to geo-potential numbers.

c) The EVRS is a zero tidal system, in agreement with the IAG resolutions.

The enhancing of the EVRS needs a revision of EVRS2000 conventions and parameters, and a new realization EVRF2007. From practical point of view, the tendency is not to change height values if not necessary. The EUREF community prefers to tie the height values to the height level of EVRF2000 and to keep normal heights. The transformation parameters between the EVRS realization and national height reference systems have to be specified to higher accuracy and topicality.

Levelling, height system definition, 3-D reference, geoid models (European, global), tidal correction, kind of height/geoid model are all related to each other. For many practical applications the choices made and parameters selected in their construction are not critical; however, it is essential that they be consistently implemented throughout.

In future the maintenance process of the EVRF2007 shall be based on a combined monitoring with several observation techniques using the approach of the European Combined Geodetic Network (ECGN), embedded in the Global Geodetic Observing System (GGOS). In addition the EUREF Permanent Network EPN can be used. Although most values behave kinematically we can make them "pseudo static" by referring them to a certain epoch.

The determination of the relationship of the European system EVRS and its realization EVRF to a Global Vertical Reference System (GVRS) and its realization Global Vertical Reference Frame (GWRF), and to the spatial reference frame ITRF is necessary for global georeferencing.

2. The European Vertical Reference System (EVRS)

The European Vertical Reference System (EVRS) is related to the Earth gravity field on and outside the solid Earth body. The EVRS is a geopotential reference system co-rotating with the Earth. In the EVRS, positions of points have geopotential values relative to the reference potential, and corresponding coordinates in a defined Terrestrial Reference System (TRS).

The European Vertical Reference Frame (EVRF) is a set of physical points with precisely determined differences of geopotential relative to a reference potential W_0 at a defined Epoch. The positions of the points are given as coordinates X in a specific spatial coordinate system attached to a Terrestrial Reference System. The EVRF is said to be a realization of the EVRS at a defined epoch. The EVRF can be realized with spirit levelling combined with gravity measurements, or using a geopotential model $W = W(X)$ where X represents the coordinates in the TRS.

We have to consider conventions both for European Vertical Reference System EVRS and its realization EVRF:

3. EVRS Definition

The European Vertical Reference System (EVRS) is a kinematical height reference system. The EVRS definitions fulfil the following four conventions:

(1) The vertical datum is defined as the equipotential surface for which the Earth gravity field potential is constant:

$$W_0 = W_{0E} = const.$$

and which is in the level of the Normaal Amsterdams Peil.

(2) The unit of length of the EVRS is the meter (SI). The unit of time is second (SI). This scale is consistent with the TCG time coordinate for a geocentric local frame, in agreement with IAU and IUGG (1991) resolutions. This is obtained by appropriate relativistic modelling.

(3) The height components are the differences ΔW_P between the potential W_P of the Earth gravity field through the considered points *P*, and the potential W_{0E} of the EVRS conventional zero level. The potential difference $-\Delta W_P$ is also designated as the geopotential number c_P

$$-\Delta W_P = c_P = W_{0E} - W_P$$

Normal heights are equivalent with geopotential numbers, provided that the reference gravity field is specified.

(4) The EVRS is a zero tidal system, in agreement with the IAG Resolutions No. 9 and 16 adopted in Hamburg in 1983 (Appendix 2).

4. EVRS Realization: EVRF2007 Principles and Strategy

EVRF2007, the new realization of the EVRS, is based on a combination strategy of three elements: the network, the vertical datum and the observation of the time evolution of the reference frame. The data is reduced, where possible, to the epoch 2000. The determination of the relationship of EVRF2007 to a global vertical reference frame (GVRS) is a future step

Network

The EVRF2007 network is realized by a new adjustment of the UELN using geopotential numbers. All participating countries were asked to contribute up-to-date data (SACHER, M., IHDE, J., SVENSSON, R., 2006; SACHER, M., IHDE, J., LIEBSCH G., MÄKINEN J., 2008). The data in the main area of the Fennoscandian Postglacial Rebound (PGR) is reduced to the epoch 2000 by differences of vertical velocities relative to the geoid from the land uplift model NKG2005LU (ÅGREN, J., SVENSSON, R., 2007). The geopotential differences are reduced into the zero tidal system.

Datum specification

In the UELN 95/98 solution that forms the backbone of the EVRF2000, the NAP level was fixed by the reference point 000A2530 in the Netherlands (not to be confused with the datum point of the national levelling network 000A2350). The Fifth precision levelling of the Netherlands does not include this reference point. So it is not possible to relate the new adjustment of UELN to this reference, which had been used since UELN-55 (SIMONSEN, O. 1960)

Classical two- and one-dimensional geodetic networks are related to one datum point, where the datum parameters give the connection to the physical Earth. On the other hand, datum specification of three-dimensional networks uses concepts that are based on globally distributed reference stations. This is a suitable opportunity to change in the EVRF the “one datum point concept” to a multi-station specification.

Therefore the UELN adjustment for the EVRF2007 is fitted to the EVRF2000 solution by choosing a number of datum points and introducing their UELN 95/98 heights into the free adjustment of the current network. For the *n* datum points P_i it is set

$$\sum_{i=1}^{n} (c_{P_i\,2007} - c_{P_i\,95/98}) = 0\,. \qquad (4\text{-}1)$$

For this transformation purpose it is important to choose stable marked datum points located in a stable part of the European plate. These datum points are part of both adjustments 2007 and 1998. The participating countries located in the stable part of the European plate were asked to propose datum points for the EVRF2007.

Influence of vertical motion and of the tidal system on datum specification

The NAP level for the EVRF2000 was defined by the reference point 000A2530 in Netherlands. In addition to possible local motion of the mark, the whole area is thought to be subsiding of the order of 1 mm/yr. In the re-definition of the NAP datum through the multiple datum points neither this movement nor the movements of the datum points themselves is corrected for.

In the condition $\sum_{i=1}^{n}(c_{P_i 2007} - c_{P_i 95/98}) = 0$ the tidal systems of the geopotential numbers must be specified. In the new UELN adjustment the geopotential differences are in the zero tidal system. While the EVRS2000 is by definition a zero-tide system, most of the data of the UELN 95/98 adjustment are not reduced for tidal effects, and the differences of its geopotential numbers $c_{P95/98}$ are related approximately to a mean tidal system. This is valid in the area of the finally selected datum points for EVRF2007. We therefore consider the differences of $c_{P_i 95/98}$ values to be in the mean tidal system.

The tidal system of the NAP datum in EVRF2000 is a separate question. It could be argued that since the NAP originally derives from sea level (the mean high tide at Amsterdam 1684), the NAP datum of EVRF2000 should be interpreted to be mean-tide. On the other hand, the EVRS2000 is defined to be zero-tide which implies that its NAP datum can be considered to be zero-tide. Since we are intent on keeping the numerical heights of the EVRF2007 close to the EVRF2000, we adopt the latter viewpoint. The condition for the datum determination thus becomes

$$\sum_{i=1}^{n}\left[c_{P_i 2007} - \left(c_{P_i 95/98} + W_2(\varphi_{P_i}) - W_2(\varphi_{NAP})\right)\right] = 0 \qquad (4\text{-}2)$$

where $c_{P_i 95/98}$ is the original EVRF2000 geopotential number of the EVRF2007 datum point P_i, $W_2(\varphi_{P_i})$ is its correction from the mean tide system to the zero tide and $W_2(\varphi_{NAP})$ is the corresponding correction at the latitude of the EVRF2000 reference point 000A2530. In this way, the geopotential numbers in EVRF2007 (zero tidal system) will be close to the geopotential numbers in EVRF2000 at the latitude of 000A2530.

The details of the EVRF2007 are treated in the companion paper by SACHER, M., IHDE, J., LIEBSCH G., MÄKINEN J., (2008).

Reference ellipsoid, normal gravity field

The EVRS2007 is defined in terms of geopotential. It is realized using geopotential numbers determined by levelling, or alternatively a geopotential model and 3-D coordinates. None of these quantities depend on ellipsoidal reference, and therefore a reference ellipsoid is not part of the EVRS definitions as long as we are only concerned with geopotential numbers. However, to convert the geopotential numbers to normal heights, a normal gravity field and geodetic latitude is required. The GRS80 normal gravity field is adopted for the purpose, evaluated at ETRS89 coordinates. The numerical values are given in Appendix 1.

Tidal corrections, permanent tide

The European Vertical Reference System EVRS deals with both geopotential and geometric quantities (station positions). Thus their variation in time, caused by geodynamical phenomena like solid earth and ocean tides is relevant for the EVRS. The IERS Conventions (MCCARTHY AND PÉTIT, 2004) in this respect cover both positions and geopotential and do not need to be duplicated for the EVRS. However they would need to be adapted for EVRS or any Vertical Reference System. E.g., the corrections to precise levelling for solid Earth tides do not probably require the detailed wave decomposition of the IERS Conventions. Or, to take another example, the IERS Conventions base the treatment of ocean loading effects in the geopotential on spherical harmonics, while for coastal effects in precise levelling a Green's function approach would probably be more convenient. It is not the purpose of the present document to develop EVRS standards in this field. In any case, it would not be practicable to reprocess UELN data according to them now. For the maintenance of the EVRF, future EVRS standards can be adapted from the future GVRS standards within the GGOS.

One particular aspect must however be addressed here, namely the treatment of the permanent tide. This is done in the following chapter.

5. The tidal system of EVRS

Background

The time-averages of the tide-generating potentials of the Sun and the Moon are not zero. At the surface of the Earth both their magnitude and their range are a few parts of 10^{-8} of the potential of the Earth. To deal with the permanent deformation they cause, there are two concepts that are available for the 3-D shape of the Earth (also called crust or topography) and three concepts for the gravity field (EKMAN, M. 1989; MÄKINEN J., IHDE J. 2008).

- In the *non-tidal* or *tide-free* system, the permanent deformation is eliminated from the shape of the Earth. From the potential field quantities (gravity, geoid, etc.) both the tide-generating potential, and the deformation potential of the Earth (the indirect effect) are eliminated. This corresponds to physically removing the Sun and the Moon to infinity. Typically the permanent deformation is treated using the same Love number h and Shida number λ for the shape, and the same Love number k for the deformation potential as for the time-dependent tidal effects, instead of estimates for secular (fluid) Love and Shida numbers. We then have the conventional tide-free system.
- In the *mean* system, the permanent effect is not removed from the shape of the Earth. The shape therefore corresponds to the long-time average under tidal forcing. The potential field retains the potential of this average Earth, and also the time-average of the tide-generating potential (though it is not due to the masses of the Earth).
- For the potential field quantities, a "middle alternative", the *zero* system is available. It eliminates the tide-generating potential but retains its indirect effect, i.e., the potential of the permanent deformation of the Earth. Thus its logical partner is the mean system for 3-D shape. In order not to need specify separately „zero for geopotential, mean for 3-D", the terminology is adopted that for the 3-D shape we have zero≡mean. In the zero system, the gravity field is generated only by the masses of the Earth (plus the centrifugal force).

The EVRS2007 is a zero-tide system, in harmony with the IAG resolutions of 1983 (Appendix 2). Therefore geopotential numbers in EVRF2007 shall express the zero-tide geopotential at the mean-tide position of the bench marks. In this chapter three issues are addressed: (i) numerical values for the permanent tide, (ii) obtaining zero-tide geopotential differences from the mixed tidal systems of national levellings in the UELN data, and (iii) taking into account the fact that 3-D coordinates in the ETRS89 and ITRFxx give the (conventional) tide-free positions.

Numerical values for the permanent tide

For second-degree tides, the time-average of the summed tide-generating potential of celestial bodies can be written in the form

$$W_2 = B\left(\frac{r}{R}\right)^2 P_2(\sin\phi) = B\left(\frac{r}{R}\right)^2\left(\frac{3}{2}\sin^2\phi - \frac{1}{2}\right) = A\left(\frac{r}{a}\right)^2\left(\sin^2\phi - \frac{1}{3}\right) \quad (5\text{-}1)$$

Here (r, ϕ) are the geocentric radius and latitude, respectively, R is a scaling factor for distance, the coefficients A or B depend on the R chosen, and P_2 (.) is the second-degree Legendre polynomial. The last form on the right is convenient for us. In it we have chosen $R=a$ where a is the semi-major axis of the GRS80 ellipsoid.

The IERS conventions of 2003 (MCCARTHY AND PÉTIT 2004, section 7.1.3) give the amplitude of W_2 in the Cartwright-Tayler-Edden normalization as H_0 = 0.31460 m. Transforming it to the form above using the parameters specified in the Section 6.5 (*op.cit.*) we have $A = -2.9166\ \mathrm{m^2s^{-2}}$.

$$W_2(r,\phi) = A\left(\frac{r}{a}\right)^2\left(\sin^2\phi - \frac{1}{3}\right), \quad A = -2.9166\,\mathrm{m^2s^{-2}} \quad (5\text{-}2)$$

The value H_0 = 0.31460 in the IERS Conventions that was used to obtain the coefficient A in Eq. (5.2) refers to the epoch 2000.0. No rate is provided. Recent high-accuracy tidal expansions (e.g., KUDRYATSEV, 2004) give a coefficient A that in the epoch 2000.0 differs less than 0.0001 $\mathrm{m^2s^{-2}}$ from the number in Eq. (5-2). The rate of A is -0.0009 $\mathrm{m^2s^{-2}}$/century, due to the rate in the obliquity of the ecliptic. The time-independent contribution of higher-order tides is below 0.0001 $\mathrm{m^2s^{-2}}$.

It is practical to have a formula for W_2 as a function of geodetic latitude φ and height above ellipsoid, close to the ellipsoidal surface. For the GRS80 ellipsoid

$$W_2(\varphi,h) = \left(1 + 0.31\times10^{-6}\,\mathrm{m^{-1}}\,h\right)\left(0.9722 - 2.8841\sin^2\varphi - 0.0195\sin^4\varphi\right) \quad [\mathrm{m^2s^{-2}}] \quad (5\text{-}3)$$

where h is the height above the ellipsoid. For the purposes of the EVRS we neglect the dependence on h. On the European topography, the error is less than 0.001 $\mathrm{m^2s^{-2}}$. We thus use

$$W_2(\varphi) = 0.9722 - 2.8841\sin^2\varphi - 0.0195\sin^4\varphi \quad [\mathrm{m^2 s^{-2}}] \qquad (5\text{-}4)$$

Eq. (5-4) gives the permanent tide-generating potential. As geopotential numbers have the opposite sign convention from potential differences, Eq. (5-4) is thus the function that should be *added* to (not subtracted from) geopotential numbers in the mean tidal system to obtain geopotential numbers in the zero tidal system.

For normal heights it is of interest to calculate $W_2(\varphi)/\gamma_0(\varphi)$, where $\gamma_0(\varphi)$ is the GRS80 normal gravity at ellipsoid.

$$H_2(\varphi) = W_2(\varphi)/\gamma_0(\varphi) = +99.40 - 295.41\sin^2\varphi - 0.42\sin^4\varphi \quad [\mathrm{mm}] \qquad (5\text{-}5)$$

Eq. (5-5) gives the function that should be added to normal heights in the mean tidal system to get normal heights in the zero tidal system. For this purpose it is approximate as the divisor strictly should not be the normal gravity at ellipsoid but the mean normal gravity at the plumb line (Eq. A–2). For a point at 5 km elevation the use of (5-5), instead of first converting the geopotential numbers from one tidal system to the other and then transforming to normal heights introduces an additional error of not more than 0.15 mm.

Converting the national data to the zero tidal system

As detailed by SACHER, M., IHDE, J., LIEBSCH G., MÄKINEN J., (2008), the national levelling data for EVRF2007 was delivered in various tidal systems, and was corrected to the zero system (mean crust over zero geoid) at the UELN computing centre. Figure 1 illustrates the process.

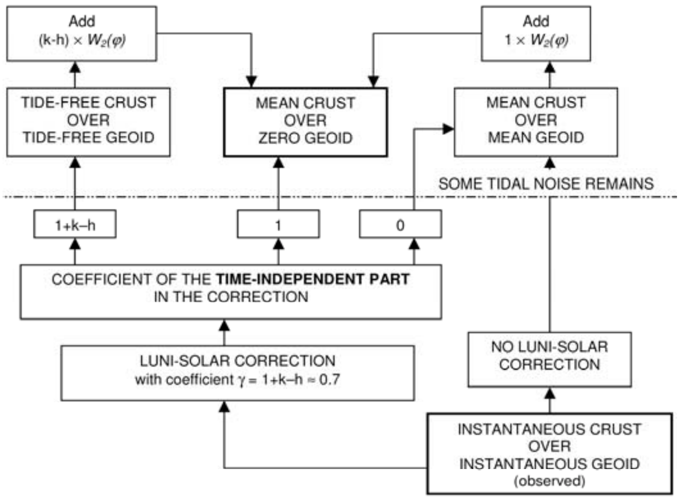

Fig 2. Flow chart for tidal correction (usually known as the luni-solar correction) to levelling as currently practised, and how to arrive at mean crust over zero geoid in the end. The parts above the dash-dot line here represent the network, the parts below represent the individual instrument setup or at most a short quickly-levelled BM interval where the time-dependent correction must be made. The role of the UELN computing centre begins at the network level. In practice the overwhelmingly most common path for the UELN data is the one on the far right. The nominal value of the coefficient γ = 1+k–h, where h and k are the Love numbers is given for illustration only.

The permanent tide in 3-D coordinates that were calculated using IERS Conventions

We wish to use 3-D coordinates of benchmarks in the ETRS89 or ITRFxx together with geopotential models to determine potential values, consistently with potential differences expressed in the zero-tidal system obtained from spirit levelling expressed in the zero-tidal system. We then must take into account that these 3-D coordinates are given in a conventional tide-free system, i.e. they give the tide-free position of the bench mark. Note that this is conceptually independent of whether the potential model in question is tide-free, zero, or mean tide.

The vector that must be to be added to the tide-free coordinates to restore the mean coordinates depends on the details of the tidal correction that was made to arrive at the tide-free coordinates in the first place. There are minor differences in the tidal correction proposed between the IERS Conventions of 1992, 1996, and 2003, with the 2003 procedure given as an alternative already in the 1996 Conventions. The restoring formulas of the 2003 Conventions (Section 7.1.3, Eqs 18a and 18b) are given with the precision of 0.1 mm. Using Eqs. 5a and 6 (Section 7.1.2, *op.cit.*) we have recalculated them with one more decimal place, to keep in line with other formulas in this chapter. The vector to be added is (Section 7.1.3, Eqs 18a and 18b, *op.cit.*)

$$\Delta\vec{r} = \{[-120.61 + 0.12\,P_2(\sin\phi)]P_2(\sin\phi)\}\,\hat{r} + \{[-25.21 - 0.06\,P_2(\sin\phi)]\sin 2\phi\}\,\hat{n} \quad [\mathrm{mm}] \qquad (5\text{-}6)$$

Here $\hat{r}$ is the unit vector from the geocenter to the station, $\hat{n}$ the unit vector to right angles to it in the northward direction, ϕ the geocentric latitude, and P_2 the second-degree Legendre polynomial. Note that in the prescriptions of the IERS Conventions (Eq. 9, Section 7.1.2, *op.cit.*) that lead to Eq. (5-6) there is no dependence on the radial position of the station.

For the sequel we need the projection of the vector $\Delta\vec{r}$ in Eq. (5-6) on the ellipsoidal normal, valid close to the surface of the GRS80 ellipsoid. We find

$$h_T(\varphi) = 60.34 - 179.01\sin^2\varphi - 1.82\sin^4\varphi \quad \text{[mm]} \tag{5-7}$$

taken positive outwards. It does not matter whether the geodetic latitude φ is calculated at the tide-free or at the mean position of the station.

6. Realization of the EVRS through a geopotential model

Calculus with total geopotential

At points *P* with accurate 3-D coordinates $X = (x, y, z)$ it will be possible to realize the EVRS by evaluating at X the geopotential W_{EGM} from a global geopotential model EGM or from a European gravimetric quasigeoid solution EGG. The W_{EGM} must provide the total (not only the disturbing) geopotential in the zero-tide system, and be given in the same reference frame (the ITRFxx or the ETRFyy) as the coordinates of *P*. A number of stations P_i, $i = 1,\dots,N$ in the EVRF2007 with 3-D coordinates $X_i = (x_i, y_i, z_i)$ in the same frame are required to determine the potential of the EVRF2007 datum in the EGM. Let c_{P_i}, $i = 1,\dots,N$ be their geopotential numbers in the EVRF2007. As ITRF and ETRF coordinates are given in the tide-free system, we must evaluate the W_{EGM}, not at the coordinates X and X_i but at the corresponding mean-tide coordinates X' and X'_i. Assuming that the observations entering the reference frame definition have been processed according to IERS Conventions 2003, the coordinates X' are obtained by $X' = X + \Delta\vec{r}$ where $\Delta\vec{r}$ is given by Eq. (5-6); similarly for the X'_i. First find $W_{0E,EGM}$, the potential of the EVRF2007 datum in the EGM (or EGG):

$$W_{0E,EGM} = \tfrac{1}{N}\sum_{i=1}^{N}\left[W_{EGM}(X'_i) + c_{P_i}\right] \tag{6-1}$$

Then using the EGM/EGG, the geopotential number of *P* is obtained as

$$c_{P,EGM} = W_{0E,EGM} - W_{EGM}(X') \tag{6-2}$$

Eq. (6-1) also provides the method to determine the difference between the EVRF2007 datum and the datum of a Global Vertical Reference Frame GVRF.

The $W_{EGM}(X')$ can approximately (to better than 0.001 m^2s^{-2}) be calculated from $W_{EGM}(X)$ through

$$W_{EGM}(X') = W_{EGM}(X) - \gamma(X)\,h_{T,P} \tag{6-3}$$

where $\gamma(X)$ is normal gravity at X and $h_{T,P}$ is the correction given by Eq. (5-7) at *P*.

Using ellipsoidal heights as an intermediate step

The calculus in the previous section using the total potential W_{EGM} is straightforward. However, as the tasks described there are often formulated in terms of ITRF or ETRF ellipsoidal heights above the GRS80 ellipsoid and of quasigeoid heights, in this section we outline the pertinent arithmetic, also to show how the correction to the ellipsoidal heights due to the tidal system emerges in this case.

As previously, let W_{EGM} be the total geopotential of an EGM or of an EGG solution in the zero-tide system. We write $W_{EGM} = U_{GRS80} + T$ where $T = W_{EGM} - U_{GRS80}$ is the disturbing potential of the W_{EGM} relative to the GRS80 normal field, containing also the zero-order terms. Let h_P and h_{P_i}, $i = 1,\dots,N$ be the ellipsoidal heights of *P* and P_i, $i = 1,\dots,N$ over the GRS80 ellipsoid, in a tide-free ETRF or ITRF frame. Now

$$\begin{aligned}
-c_{P,EGM} &= W_{EGM}(X') - W_{0E,EGM} \\
&= \left[W_{EGM}(X') - W_{EGM}(X)\right] + \left[W_{EGM}(X) - U_{GRS80}(X)\right] + \left[U_{GRS80}(X) - U_{0,GRS80}\right] + \left[U_{0,GRS80} - W_{0E,EGM}\right] \quad (6\text{-}3) \\
&= -\gamma(X)\,h_{T,P} + T(X) - \bar{\gamma}(X)\,h_P + U_{0,GRS80} - W_{0E,EGM}
\end{aligned}$$

Here $\gamma(X)$ is the normal gravity at coordinates X of *P*, $h_{T,P}$ is the correction of Eq. (5-7) at *P*, $\bar{\gamma}(X)$ is the mean value of the normal gravity between *P* (with the coordinates X) and the GRS80 ellipsoid, and h_P is the ellipsoidal height of *P* using the (tide-free) coordinates X. We have $c_{P,EGM} = \bar{\gamma}_H H_{P,EGM}$ where $H_{P,EGM}$ is the sought-for normal height of *P* in the EVRS realization using the EGM, and $\bar{\gamma}_H$ is the usual mean normal gravity for converting geopotential number to normal height. The disturbing potential $T(X)$ can be written $T(X) = \gamma_Q \varsigma_P$, where ς_P is the quasigeoid height from the EGM at *P* relative to the GRS80 normal field and γ_Q is the normal gravity at the telluroid point *Q*. The γ_Q can be replaced by $\gamma(X)$ or by $\gamma(X')$.

$$-\bar{\gamma}_H\, H_{P,EGM} = \gamma(X)\varsigma_P - \gamma(X)\, h_{T,P} - \bar{\gamma}_h\, h_P + U_{0,GRS80} - W_{0E,EGM} \qquad (6\text{-}4)$$

Neglecting the differences between the γ values ($\bar{\gamma} = \bar{\gamma}_H \approx \bar{\gamma}_h \approx \gamma(X)$) and putting $W_{0E,EGM} - U_{0,GRS80} = \bar{\gamma}\, h_{offset}$

$$-\bar{\gamma}\, H_{P,EGM} = \bar{\gamma}\,\varsigma_P - \bar{\gamma}\, h_{P,T} - \bar{\gamma}\, h_P - \bar{\gamma}\, h_{offset} \qquad (6\text{-}5)$$

$$H_{P,EGM} = h_P + h_{T,P} - \varsigma_P + h_{offset} \qquad (6\text{-}6)$$

The parameter $h_{offset} = \left(W_{0E,EGG} - U_{0,GRS80}\right)/\bar{\gamma}$ is implied by the offset between the GRS80 normal potential at ellipsoid and the potential $W_{0E,EGM}$ of the EVRF2007 datum in the EGM (Eq. 6-1). Using the reasoning above

$$W_{0E,EGM} - U_{0,GRS80} = \tfrac{1}{N}\sum_{i=1}^{N}\left[W_{EGM}(X'_i) + c_{P_i}\right] - U_{0,GRS80} = \tfrac{1}{N}\sum_{i=1}^{N}\bar{\gamma}_i\left[H_{P_i} + \varsigma_{P_i} - h_{T,P_i} - h_{P_i}\right] \qquad (6\text{-}7)$$

The H_{P_i} in Eq. (6-7) are the normal heights of the P_i, $i = 1,...,N$ in EVRF2007, and the rest of the notation is obvious.

7. Development of a Global Vertical Reference System

Under consideration of the requirements of the IAG project GGOS (RUMMEL ET AL., 2002), a global unique vertical reference system is necessary (IHDE, J., SÁNCHEZ, L., 2005). In 2003 the IAG installed the Inter-Commission Project 1.2. (ICP1.2) “Vertical Reference Frames” of Commission 1 and Commission 2. On the basis of classical and modern observations, the ICP1.2 studies the consistent modelling of both geometric and gravimetric parameters to provide the fundamentals for the adoption of a Global Vertical Reference System and its realization, a unifiedGlobal Vertical Reference Frame GVRF. (IHDE, J. 2007 ; IHDE, J. ET AL., 2007)

For global geopositioning, the transformations between the EVRF2007 and the GVRF will be provided. Their datum difference can be determined as described in Chapter 6 (Eq. 6-1). Suppose the GVRF datum is defined by adopting a reference value W_{0G} of the geopotential, in conjunction with a conventional EGM W_{EGM}. Then using the notation of Chapter 6, the datum difference is

$$W_{0E} - W_{0G} = \tfrac{1}{N}\sum_{i=1}^{N}\left[W_{EGM}(X'_i) + c_{P_i}\right] - W_{0G} \qquad (7\text{-}1)$$

8. Time evolution of the EVRF

The future time evolution of EVRF against W_{0E} and the evolution of the W_{0E} itself will be monitored by the observation of time series of the European Combined Geodetic Network (ECGN), embedded in GGOS (RUMMEL ET AL., 2002), as a carrier network of the EVRF and its datum.

The European Combined Geodetic Network (ECGN) is an integrated geodetic network for spatial reference and gravity**.** In this project time series of geometric positions (GNSS) and superconducting gravimeter measurements are combined with periodic levellings and absolute gravity observations. (IHDE, J., ET AL. 2005a, 2005b)

Objectives of ECGN are the maintenance of the long time stability of the terrestrial reference frame especially for the height component, and contribution to the European geoid modelling.

It allows the combination of various geodetic methods: repeated precise levelling, permanent GNSS, repeated absolute gravity, superconducting gravimeter measurements, satellite gravimetry, etc., to determine the time evolution of station positions and of the geopotential in Europe.

Currently 74 ECGN stations are proposed by 21 countries, though all stations do not yet fulfil the standards of ECGN. Information about the current ECGN status is available at www.bkg.bund.de/ecgn/ .

9. Normative references

The EVRS conventions are aligned to the IERS 2003 Conventions.

The following documents are indispensable for the application of this document. For dated references, only the cited edition applies. For undated references, the latest edition of the referenced document (including any amendments) applies.

- *ISO 1000, SI units and recommendations for use of their multiples and of certain other units*
- *ISO 19100 Geographic Information standard family as far as useful*
- *Conventions for the Definition and Realization of a Conventional Vertical Reference System (CVRS)*
- *EVRS2000 conventions.*

Literature

ÅGREN, J., SVENSSON, R., 2007. *Postglacial land uplift model and system definition for the new Swedish height system RH 2000.* LMV-rapport 2007:4, Gävle 2007. 124 p.

EKMAN, M., 1989. *Impacts of geodynamic phenomena on systems for height and gravity*. Bull. Géod. 63, 281–296.

IHDE, J., 2007. *Realization of a Global Vertical Reference System 2007.* In: Proceedings of the 1st IGFS Symposium Gravity Field of the Earth 28 August – 1 September 2006, Istanbul, Harita Dergisi, Special Issue 18, 2007.

IHDE J., AUGATH W., 2000. *The Vertical Reference System for Europe*; In: Veröff. Bayer. Komm. Int. Erdmess., Bayer. Akad. d. Wiss., Astron.-geod. Arb., München, 2000, H. 61, p. 99-110.

IHDE, J., AUGATH, W., 2002. *The European Vertical Reference System (EVRS), Its relation to a World Height System and to the ITRS.* Proceedings of the IAG 2001 Scientific Assembly, Budapest, Hungary, IAG Symposia, Vol. 125, 78–83, Springer Berlin.

IHDE, J., BAKER, T., BRUYNINX, C., FRANCIS, O., AMALVICT, M., KENYERES, A., MÄKINEN, J., SHIPMAN, S., ŜIMEK, J., WILMES, H., 2005.1. *Development of a European Combined Geodetic Network (ECGN).* In: Journal of Geodynamics 40, 450–460.

IHDE, J., BAKER, T., BRUYNINX, C., FRANCIS, O., AMALVICT M., LUTHARDT, J., LIEBSCH, G., KENYERES, A., MAKINEN, J., SHIPMAN, S., ŜIMEK, J., WILMES, H., 2005.2. *The Implementation of the ECGN Stations – Status of the 1st Call for Participation.* In: EUREF Publication No. 14, Mitteilungen des Bundesamtes für Kartographie und Geodäsie, Band 35, 49–58.

IHDE, J., SANCHEZ, L., 2005. *A unified global height reference system as a basis for IGGOS.* Journal of Geodynamics 40, 400–413.

IHDE, J. ET AL., 2007. *Conventions for the Definition and Realization of a Conventional Vertical Reference System (CVRS, Draft 3.0, 2007-06-15.* In: Final Report of the IAG Inter-Commission Project ICP1.2 Vertical Reference Frames to the IAG Executive Committee, presented at the IUGG General Assembly 2007 in Perugia, Italy.

KUDRAYATSEV, S.M., 2004. *Improved harmonic development of the Earth tide-generating potentia*l. Journal of Geodesy 77: 829–838.

MÄKINEN, J., IHDE, J. 2008. *The permanent tide in height systems*. In: M.G. Sideris (ed), Observing our Changing Earth. Proceedings of the 2007 IAG General Assembly, Perugia, Italy, July 2–13, 2007. IAG Symposia 133, 81–87.

MCCARTHY, D.D., ed., 1992. *IERS Standards.* IERS Technical Note 13, Observatoire de Paris, Paris.

MCCARTHY, D.D., ed.,1996. *IERS Standards*. IERS Technical Note 21, Observatoire de Paris, Paris.

MCCARTHY, D.D., PETIT, G., eds., 2004. *IERS Conventions (2003)*. *IERS Technical Note* 32. Verlag des Bundesamts für Kartographie und Geodäsie, Frankfurt am Main.

MORITZ, H., 1980. *Geodetic Reference System 1979*. Bull. Géod. 54, 395–405.

RAPP, R.H., 1983. *Tidal gravity computations based on recommendations of the Standard Earth Tide Committee.* Bull. Inf. Marées Terrestres 89, 5814–5819.

RUMMEL, R., HECK, B., 2001. *Some critical remarks on the definition and realization of the EVRS.* In: Veröff. Bayer. Komm. Int. Erdmess., Bayer. Akad. d. Wiss., Astron.-geod. Arb., München, 2000, H. 61, p. 114–115.

RUMMEL, R., DREWES, H., BEUTLER, G., 2002. *Integrated Global Geodetic Observing System (IGGOS): A candidate IAG project.* Proceedings of the IAG 2001 Scientific Assembly, Budapest, Hungary, IAG Symposia, Vol. 125, 609614, Springer, Berlin.

SACHER, M., IHDE, J., SVENSSON, R., 2006. *Status of UELN – Steps on the way to EVRS2007.* Report on the Symposium of the IAG Subcommission for Europe (EUREF), Riga, 14–17 June 2006. Mitteilungen des Bundesamtes für Kartographie und Geodäsie, Frankfurt am Main.

SACHER, M., IHDE, J.; LIEBSCH, G., MÄKINEN, J., 2008. *EVRF2007 as Realization of the European Vertical Reference System.* Presented at the Symposium of the IAG Subcommission for Europe (EUREF), London, 18–21 June 2008. Submitted to the Proceedings.

SIMONSEN, O., 1960. *Short report of the history and establishment of United European Levelling Net and on the results obtained up to December 1960*. Trav. Assoc. Int. Géod. 21, 115–158.

TSCHERNING, C.C., ed., 1984. *The Geodesist's Handbook.* Bull. Géod. 58:3.

Appendix 1. Gravity formulas

Normal gravity at the ellipsoid is computed from the Gravity Formula 1980 (MORITZ H., 1980) using the series expansion

$$\begin{aligned}\gamma_0 = 9.783\,267\,715 \big(1 &+ 0.005\,279\,0414 \sin^2\varphi \\ &+ 0.000\,023\,2718 \sin^4\varphi \\ &+ 0.000\,000\,1262 \sin^6\varphi \\ &+ 0.000\,000\,0007 \sin^8\varphi\big)\ \mathrm{m\,s^{-2}}\end{aligned} \qquad \text{(A-1)}$$

where the latitude φ is in ETRS89. The normal heights H_P were computed by $H_P = c_P / \bar{\gamma}$ where $\bar{\gamma}$ is the average value of the normal gravity along the normal plumb line between the ellipsoid and the telluroid. The average value of the normal gravity along the normal plumb line is determined by the formula

$$\bar{\gamma} \approx \bar{\gamma}_H = \gamma_0 \left[1 - \left(1 + f + m - 2f \sin^2\varphi\right)\frac{H}{a} + \frac{H^2}{a^2} \right] \qquad \text{(A-2)}$$

where H is an approximate value for H_P and γ_0 is from Eq. (A-1). The notation and the numerical values for the other quantities are according to (MORITZ H., 1980)

Appendix 2. Permanent tide – IAG resolutions

The current recommendations are the IAG resolutions adopted at the XVIII General Assembly of the IUGG in Hamburg in 1983. They are as follows (from TSCHERNING, C.C., 1984):

Resolution No. 9

The International Association of Geodesy

recognizing the high level of accuracy of both absolute and relative gravity measurements recently attained

considering the necessity to adopt standard corrections to gravity observation in order to allow intercomparisons between measurements at different epochs of time

recommends

that the tidal correction applied to the gravity observations follow the final recommendations of the Standard Earth Tide Committee as presented at the XVIII IUGG General Assembly [the Committee recommended zero tidal gravity (RAPP, R.H., 1983), note added by the authors]

Resolution No. 16

The International Association of Geodesy

recognizing the need for the uniform treatment of tidal corrections for various geodetic quantities such as gravity and station positions, and

considering the reports of the Standard Earth Tide Committee and S.S.G. 2.55, Predictive Methods for Space Techniques, presented at the XVIII General Assembly,

recommends that:

the indirect effect due to the permanent yielding of the Earth be not removed.